

# ON AN OPEN QUESTION CONCERNING PRODUCT-TYPE DIFFERENCE EQUATIONS

JULIUS FERGY T. RABAGO

ABSTRACT. In [Acta Math. Univ. Comenianae Vol. LXXX, 1 (2011), pp. 63–70], Yang, Chen and Shi examined the system of difference equations

$$x_n = \frac{a}{y_{n-p}}, \quad y_n = \frac{by_{n-p}}{x_{n-q}y_{n-q}}, \quad n = 0, 1, \dots,$$

where  $q$  is a positive integer with  $p < q$ ,  $p \nmid q$ ,  $p \geq 3$  is an odd number, both  $a$  and  $b$  are nonzero real constants, and the initial values  $x_{-q+1}, x_{-q+2}, \dots, x_0, y_{-q+1}, y_{-q+2}, \dots, y_0$  are nonzero real numbers. At the end of their note, they posted a question regarding the behaviour of solutions of the given system when  $p$  is even. More precisely, they asked what the solutions of the system may look like if  $p$  is even. In this note we answer this question raised by the authors. Particularly, we show that the system may or may not admit a periodic solution depending on the coprimality of the parameters  $p$  and  $q$  and on the parity of the integer  $p/\gcd(p, q)$ .

## 1. INTRODUCTION

In a recent paper, Yang, Chen and Shi investigated the system of difference equation

$$(1) \quad x_n = \frac{a}{y_{n-p}}, \quad y_n = \frac{by_{n-p}}{x_{n-q}y_{n-q}}, \quad n = 0, 1, \dots,$$

where  $q$  is a positive integer with  $p < q$ ,  $p \nmid q$ ,  $p \geq 3$  is an odd number, both  $a$  and  $b$  are nonzero real constants, and the initial values  $x_{-q+1}, x_{-q+2}, \dots, x_0, y_{-q+1}, y_{-q+2}, \dots, y_0$  are nonzero real numbers. They have showed in [11] that all real solutions of the system are eventually periodic with period  $2pq$  (resp.  $4pq$ ) when  $(a/b)^q = 1$  (resp.  $(a/b)^q = -1$ ). Further, the authors [11] characterized the asymptotic behavior of the solutions of (1) when  $a \neq b$ . Their work is actually a generalization of a result seen in [10], wherein Özban investigated the behavior of the positive solutions of the system of rational difference equations

$$(2) \quad x_n = \frac{a}{y_{n-3}}, \quad y_n = \frac{by_{n-3}}{x_{n-q}y_{n-q}}, \quad n = 0, 1, \dots,$$

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where  $q > 3$  is a positive integer with  $3 \nmid q$ ,  $a$  and  $b$  are positive constants. Özban particularly showed in [10] that for the case  $b = a$ ,  $p = 3$ ,  $q > 3$ , and  $p$  not dividing  $q$ , all positive solutions of the system (1) are eventually periodic with period  $6q$ . Meanwhile, Özban's work [10] was inspired by a result found by Yang, Liu and Bai in [12]. In their four page note, Yang et al. [12] investigated the system (1) for the case where  $p$  and  $q$  are positive integers with  $p \leq q$ , and  $a$  and  $b$  are positive constants. They have showed that if  $b = a$ , and  $q$  is divisible by  $p$ , then every positive solution of (1) is periodic with period  $2q$ . In [6], an intriguing result regarding the behavior of positive solutions of the higher-order difference equation

$$(3) \quad x_n = \frac{cx_{n-p}x_{n-p-q}}{x_{n-q}}, \quad n = 0, 1, \dots,$$

where  $p, q \in \mathbb{N}$  and  $c > 0$ , was obtained by Iričanin and Liu through an elegant and short way. One particular results they presented in (3) reads as follows: if  $c = 1$  in (3) and  $\gcd(p, q) = 1$ , and  $p$  is odd, then all positive solutions of (3) are eventually periodic with period  $2pq$ . Their result was in fact inspired by earlier results presented in [9, 10] and [12]. Similar nonlinear systems of rational difference equations were also investigated, see, e.g., Cinar [1], Cinar and Yalçinkaya [2, 3, 4], Cinar et al. [5], and Özban [8].

Now, our aim in this work is to answer a question raised by Yang et al. at the end of their paper [11]. Particularly, we shall described here the behavior of solutions of (1) in the case when  $p$  is even. We mention that Yang et al. [11] already made a preliminary observation on the behavior of solutions of (1) when  $p$  is even. More specifically, they have observed, after some numerical experimentations, that (1) is non-periodic when  $p$  is even. Here we show, through an analytical approach, that this observation is in fact true whenever  $\gcd(p, q) = 1$ , i.e.  $q$  is odd and that, in addition, every positive solution of (1) when  $b = a$  has an exponentially growing/decaying subsequence irrespective of the choice of positive initial values  $x_{-q+1}, x_{-q+2}, \dots, x_0, y_{-q+1}, y_{-q+2}, \dots, y_0$  on this case. Every solution, however, will be periodic of period  $m$ , where  $m$  denotes the least common multiple of  $p$  and  $2q$ , if  $p/\gcd(p, q)$  is odd. On the other hand, if  $p/\gcd(p, q)$  is even, then every solution of (1) behaves in a similar fashion as in the case when  $\gcd(p, q) = 1$ . That is, there is some subsequence of the solution  $\{x_n\}$  (resp.  $\{y_n\}$ ) which tends to infinity (resp. converges to zero) (cf. Theorem 1). We emphasize that the problem raised by Yang et al. in [11] remains open since Iričanin and Liu [6] only deal with the case when  $p$  is odd in (1), which is not of interest here.

**Remark 1.** By an eventually periodic solution  $\{(x_n, y_n)\} := \{(x_n, y_n)\}_{n=-(q-1)}^\infty$  of (1), we mean that there exist an integer  $n_0 \geq -q+1$  and a positive integer  $\pi$  such that

$$(x_{n+n_0+\pi}, y_{n+n_0+\pi}) = (x_{n+n_0}, y_{n+n_0}), \quad n = 1, 2, \dots,$$

and  $\pi$  is called a period (cf. [7]). If, regardless of the choice of initial values  $x_{-q+1}, x_{-q+2}, \dots, x_0, y_{-q+1}, y_{-q+2}, \dots, y_0$ , no such values of  $n_0$  and  $\pi$  exist, then every solution of (1) is not and can never be periodic.

## 2. MAIN RESULTS

In this section we show that the system (1) can never have a periodic or eventually periodic solution when  $p$  is even and  $q$  is odd. If, however,  $q$  is even, then the existence of periodic solution of (1) depends on the parity of  $p/\gcd(p, q)$ . Our approach parallels that seen in [6].

We only consider the case when  $b = a$  in (1) with all of its initial values taken from the set of positive real numbers. The same inductive lines, however, can be followed to show a similar result for the case  $b = -a$  and even for the more general case given by system (1).

To show that (1) has no periodic solution, it suffices to prove that every solution of (1) has an increasing subsequence (or, perhaps, a decreasing subsequence) regardless of the choice of initial values  $x_{-q+1}, x_{-q+2}, \dots, x_0, y_{-q+1}, y_{-q+2}, \dots, y_0$ .

With this idea in mind, we now proceed as follows. First, we transform the first equation in (1) to

$$x_n x_{n-q} = x_{n-p} x_{n-p-q}, \quad n = 0, 1, \dots$$

Since  $x_n > 0$  for all  $n \geq 0$ , then taking the natural logarithm of both sides of the above equation and making the substitution  $a_n := \ln x_n$ , we get

$$a_n + a_{n-q} - a_{n-p} - a_{n-p-q} = 0.$$

Using the ansatz  $a_n = \lambda^n \in \mathbb{R}$ , we obtain the polynomial equation

$$P(\lambda) := \lambda^{p+q} + \lambda^p - \lambda^q - 1 = (\lambda^p - 1)(\lambda^q + 1) = 0.$$

From here on, we consider two possibilities: (i)  $\gcd(p, q) = 1$ ; and (ii)  $\gcd(p, q) > 1$ .

CASE 1: Suppose that  $\gcd(p, q) = 1$  or equivalently,  $q$  is odd. Then, it is evident that  $\lambda = -1$  is a root of  $P(\lambda) = 0$  of order two. Denote this repeated root by  $\lambda_1$  and  $\lambda_{p+1}$ , i.e., let  $\lambda_1 = \lambda_{p+1} = -1$ . Then, the explicit formula for the sequence  $\{a_n\}$  is of the form

$$a_n = c_1 \lambda_1^n + c_{p+1} n \lambda_{p+1}^n + \sum_{\substack{i=2 \\ i \neq p+1}}^{p+q} c_i \lambda_i^n,$$

for some real numbers  $c_1, c_2, \dots, c_{p+q}$ . Note that  $P(\lambda) = 0$  has all of its roots on the unit disk  $|\zeta| \leq 1$ . Denote these roots by  $\{\lambda_i\}_{i=1}^{p+q}$  where  $\{\lambda_i\}_{i=1}^p$  are the corresponding roots of  $\lambda^p - 1 = 0$  and  $\{\lambda_i\}_{i=p+1}^{p+q}$  are the roots of  $\lambda^q + 1 = 0$ . Clearly,  $\lambda_i^p = 1$  for all  $1 \leq i \leq p$  and  $\lambda_i^{2q} = 1$  for all  $p+1 \leq i \leq p+q$ . Since

$p \nmid q$ , then  $\lambda_i^{2pq} = 1$  for all  $1 \leq i \leq p+q$ . Hence,

$$a_{2pqn} = c_1 + 2c_{p+1}pqn + \sum_{\substack{i=2 \\ i \neq p+1}}^{p+q} c_i.$$

Suppose  $c_{p+1} > 0$ , then for sufficiently large  $N \in \mathbb{N}$ ,  $a_{2pqn}$  will eventually be increasing for  $n \geq N$ , in fact we'll have

$$a_{2pqn} \longrightarrow \infty \quad \text{as} \quad n \longrightarrow \infty.$$

Going back to the relation  $a_n = \ln x_n$ , we see that

$$x_{2pqn} = \exp\{a_{2pqn}\} \longrightarrow \infty \quad \text{as} \quad n \longrightarrow \infty.$$

Moreover, we have

$$y_{2pqn} \longrightarrow 0 \quad \text{as} \quad n \longrightarrow \infty.$$

Therefore, the subsequence  $\{x_{2pqn}\}$  (resp.  $\{y_{2pqn}\}$ ) will tend to infinity (resp. converges to zero) exponentially. Thus, every solution of (1) can never be periodic when  $\gcd(p, q) = 1$ , or equivalently, when  $q$  is odd.

CASE 2: Now, the case  $\gcd(p, q) > 1$  needs a little more work. Suppose  $\gcd(p, q) = 2^r u$  for some odd integer  $u \geq 1$  and integer  $r > 0$ . Then, the roots of  $P(\lambda) = 0$  can be expressed as

$$(4) \quad \begin{cases} \exp\left\{\frac{(2k+1)\pi i}{q}\right\}, & k = 0, 1, \dots, q-1, \\ \exp\left\{\frac{2l\pi i}{p}\right\}, & l = 0, 1, \dots, p-1. \end{cases}$$

Let  $p = 2^r us$  and  $q = 2^r ut$  where  $s < t$  and  $s \nmid t$ . The roots of  $P(\lambda) = 0$  are simple if and only if

$$\frac{2k+1}{2^r ut} \neq \frac{2l}{2^r us}, \quad \text{for each } k, l \in \mathbb{N}_0,$$

or equivalently

$$(5) \quad (2k+1)s \neq 2lt, \quad \text{for each } k, l \in \mathbb{N}_0.$$

We consider two separate subcases, namely: (a)  $p/\gcd(p, q)$  is odd; and (b)  $p/\gcd(p, q)$  is even.

Subcase 2.1: Clearly, if  $s$  is odd (i.e.,  $p/\gcd(p, q)$  is odd), then the inequality (5) always holds. Hence, the roots of (1) are distinct. Moreover, since  $\lambda^{2q} = 1$ , then the explicit formula for  $a_{mn}$  takes the form

$$a_{mn+t} = \sum_{i=1}^{p+q} c_i \lambda_i^{mn+t} = \sum_{i=1}^{p+q} c_i \lambda_i^t = a_t, \quad \forall n = 0, 1, \dots,$$

for each  $t = \{0, 1, \dots, m-1\}$ , where  $m := \text{lcm}(p, 2q)$  denotes the least common multiple (lcm) of  $p$  and  $2q$ . Therefore,  $a_n$  is eventually periodic with

period  $m$ . Since  $a_n = \ln x_n$  for all  $n = 0, 1, \dots$ , then  $x_n$ , as well as  $y_n$ , are also eventually periodic.

Subcase 2.2: Now, if  $p/\gcd(p, q)$  is even (i.e.,  $s = 2^{r_0}s_0 < t$  for some integers  $r_0, s_0 > 0$  with  $s_0$  being odd), then  $t$  must be odd, otherwise  $\gcd(p, q) > 2^r u$ . Evidently, (5) does not hold since the equality  $(2k+1)2^{r_0-1}s_0 = lt$  may hold true by choosing appropriate values for  $l, k \in \mathbb{N}_0$ . For instance, if  $r_0 = 1$ , then we can choose  $l = s_0$  and  $k = (t-1)/2$ . In general, we can take  $l = 2^{r_0-1}s_0$  and  $k = (t-1)/2$ . Since the inequality was not satisfied, then  $P(\lambda) = 0$  has at least one repeated root. Without loss of generality, let  $\lambda_j$  be a root of  $P(\lambda) = 0$  of order two and  $m$  be the least common multiple of  $p$  and  $2q$ . By arguing as in the first case, we see that the subsequence

$$\begin{aligned} a_{mn} &= c_j mn \lambda_j^{mn} + \sum_{i=1}^{p+q} c_i \lambda_i^{mn} = c_j mn + \sum_{i=1}^{p+q} c_i \\ &\longrightarrow \infty \quad \text{as } n \longrightarrow \infty. \end{aligned}$$

Again, going back to the relation  $a_n = \ln x_n$ , the above result leads us to conclude that when  $\gcd(p, q) > 1$  and  $p/\gcd(p, q)$  is even, a subsequence of the solution to (1) grows/decays exponentially. Thus, every solution in this case is non-periodic.

We remark that every solution to (1) when  $b = -a$  has the same behavior as those in the case when  $b = a$ . The only difference is that every real solution of (1) for  $b = -a$  oscillates at 0. This can be seen easily from the relation  $x_n x_{n-q} = -x_{n-p} x_{n-p-q}$ . In fact, if  $\{(x_n, y_n)\}$  is a solution to (1) with positive initial values, then there is some subsequence  $\{|x_{mn+t}|\}$  (resp.  $\{|y_{mn+t}|\}$ ) which tends to infinity (resp. converges to zero) exponentially.

We summarize our discussion in the following theorem for the case  $b = a$ . A similar conclusion can be established for  $b = -a$ .

**Theorem 1.** *Let  $\{(x_n, y_n)\}$  be a solution to (1). Then, the the following statements are true:*

- (i) *If  $\gcd(p, q) = 1$ , then the solution  $\{(x_n, y_n)\}$  of system (1) has a subsequence  $\{x_{2pqn}\}$  (resp.  $\{y_{2pqn}\}$ ) that tends to infinity (resp. converges to zero) exponentially, and vice versa.*
- (ii) *If  $\gcd(p, q) > 1$ , and  $p/\gcd(p, q)$  is odd, then the solution  $\{(x_n, y_n)\}$  of system (1) is eventually periodic with period  $m$ , where  $m$  denotes the least common multiple of  $p$  and  $2q$ .*
- (iii) *If  $\gcd(p, q) > 1$  and  $p/\gcd(p, q)$  is even, then the solution  $\{(x_n, y_n)\}$  behaves in a similar fashion as in (i).*

Some illustrations which shows different behaviors of various solutions of system (1) for the case  $b = a$  with random positive initial values are illustrated in Figures (1)–(5).

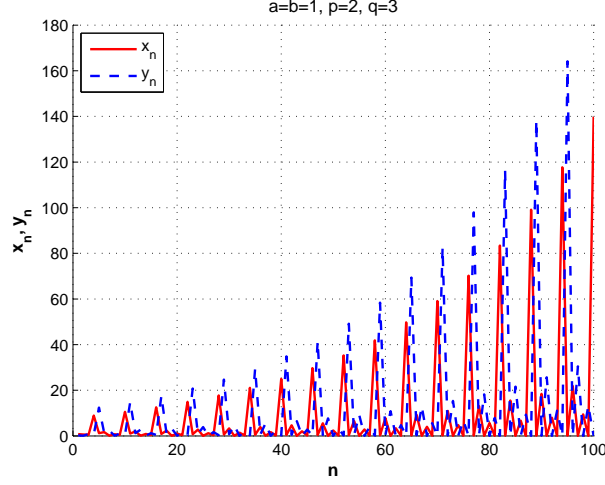


FIGURE 1. It is noticeable from the above illustration that the system of difference equation

$$x_n = \frac{1}{y_{n-2}}, \quad y_n = \frac{y_{n-2}}{x_{n-3}y_{n-3}}, \quad n = 0, 1, \dots$$

has a solution  $\{x_n\}$  (resp.  $\{y_n\}$ ) with a subsequence  $\{x_{6n+s}\}$  (resp.  $\{y_{6n+t}\}$ ) ( $0 \leq s, t < 6$ ) that tends to infinity. This result, moreover, agrees with Theorem 1–(i)

**Remark 2.** We remark that a similar system has been studied by Özban in [9] wherein he investigated the behavior of positive solutions of the system

$$x_{n+1} = \frac{1}{y_{n-k}}, \quad y_{n+1} = \frac{y_{n-k}}{x_{n-m}y_{n-m-k}}, \quad n = 0, 1, \dots,$$

where  $k$  is a nonnegative integer and  $m$  is a positive integer. His main result states that all solutions of the above system of difference equations are periodic with period  $2m + 2k + 2$ . In particular, when  $b = a$  and  $k = 0$ , all solutions of above equation is periodic with period  $2q + 2$ .

### 3. CONCLUSION

In this short note, we have investigated the behavior of positive solutions of the system

$$x_n = \frac{a}{y_{n-p}}, \quad y_n = \frac{by_{n-p}}{x_{n-q}y_{n-q}}, \quad n = 0, 1, \dots,$$

for  $b = a$ , where  $q$  is a positive integer with  $p < q$ ,  $p \nmid q$ , and  $p$  is an even number. We have found that every solution of the above system when  $b = a$ , with  $p > 0$ , is non-periodic and has a subsequence that grows/decays exponentially whenever  $q$  is odd. However, a periodic solution of the given system occurs when  $\gcd(p, q) > 1$  and  $p/\gcd(p, q)$  is odd. In this case, the

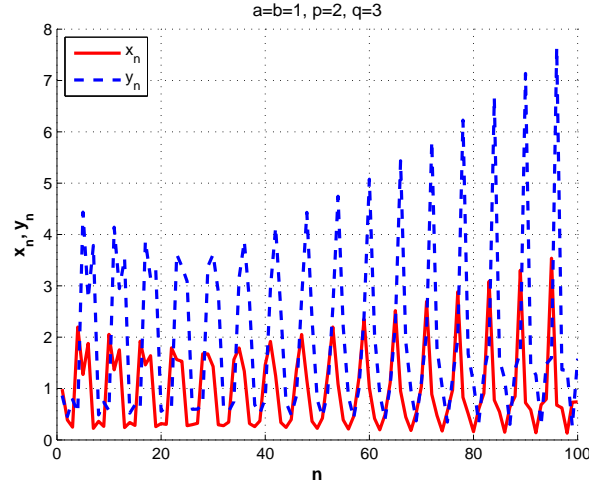


FIGURE 2. The above figure illustrates the behavior of another solution of the system

$$x_n = \frac{1}{y_{n-2}}, \quad y_n = \frac{y_{n-2}}{x_{n-3}y_{n-3}}, \quad n = 0, 1, \dots$$

Observe that after  $N = 25$ , there is some subsequences  $\{x_{6n+s}\}$  and  $\{y_{6n+t}\}$  which both tends to infinity for  $n \geq N$

period of the solution appears to be equal to the least common multiple of  $p$  and  $2q$ . On the other hand, a similar behavior as for the case when  $q$  is odd was observed when  $\gcd(p, q) > 1$  and  $p/\gcd(p, q)$  is even. Consequently, our result settled the question raised by Yang et al. in [11] about the behavior of solution of the given equation on the case when  $p$  is even and  $q > p$  in the given system.

#### APPENDIX

From the polynomial equation  $P(\lambda) = (\lambda^p - 1)(\lambda^q + 1) = 0$ , it is clear that  $\lambda = 1$  is a simple root. Hence, a particular solution to the non-homogeneous equation

$$(6) \quad a_n + a_{n-q} - a_{n-p} - a_{n-p-q} = \ln c, \quad c := a/b,$$

has the form

$$a_n^p = An$$

from which, by simple computation, leads to  $A = \ln(c/2p)$ . Thus, if  $\gcd(p, q) > 1$  and  $p/\gcd(p, q)$  is odd, then the general solution of equation (6) takes the

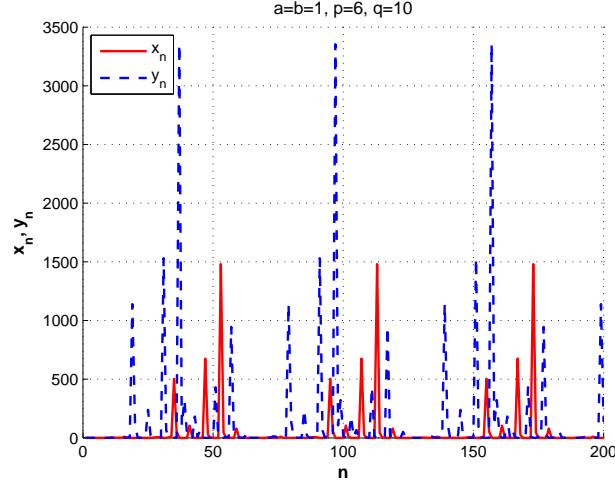


FIGURE 3. The above figure shows an interesting behavior of solutions of the system

$$x_n = \frac{1}{y_{n-6}}, \quad y_n = \frac{y_{n-6}}{x_{n-10}y_{n-10}}, \quad n = 0, 1, \dots$$

Clearly, as the above system satisfies the conditions in Theorem 1–(ii), we then have a periodic solution of period  $\text{lcm}(6, 2 \times 10) = 60$

form

$$x_n = e^{a_n} = c^{n/2p} \exp \left\{ \sum_{l=0}^{p-1} \left( \alpha_{l,1} \cos \frac{2l\pi n}{p} + \alpha_{l,2} \sin \frac{2l\pi n}{p} \right) + \sum_{k=0}^{q-1} \left( \beta_{k,1} \cos \frac{(2k+1)\pi n}{q} + \beta_{k,2} \sin \frac{(2k+1)\pi n}{q} \right) \right\}.$$

We can write  $x_n$  as  $x_n = c^{n/2p} \hat{x}_n$ , where  $\hat{x}_n$  denotes the positive solution of equation (6) with  $c = 1$ .

From above discussion, together with Theorem 1–(ii), we get the following results.

**Theorem 2.** *Assume that  $c \in (0, 1)$ ,  $\gcd(p, q) > 1$  and  $p/\gcd(p, q)$  is odd and let  $m = \gcd(p, 2q)$ , then every positive solution of (6) converges geometrically to zero. Moreover, for each  $t \in \{0, 1, \dots, m-1\}$ , the subsequence  $\{x_{mn+t}\}_{n \in \mathbb{N}_0}$  converges monotonically to zero as  $n$  tends to infinity.*

**Theorem 3.** *Assume that  $c > 1$ ,  $\gcd(p, q) > 1$  and  $p/\gcd(p, q)$  is odd and let  $m = \gcd(p, 2q)$ , then every positive solution of (6) tends to infinity. Moreover, for each  $t \in \{0, 1, \dots, m-1\}$ , the subsequence  $\{x_{mn+t}\}_{n \in \mathbb{N}_0}$  grows exponentially as  $n$  tends to infinity.*



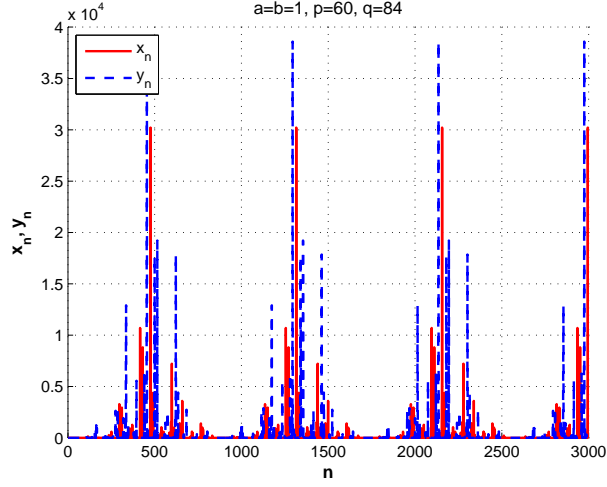


FIGURE 4. Another interesting illustration for Theorem 1–(ii) is shown above. In this example, the system

$$x_n = \frac{1}{y_{n-60}}, \quad y_n = \frac{y_{n-60}}{x_{n-84}y_{n-84}}, \quad n = 0, 1, \dots,$$

has been considered. A simple computation for the period  $m$  of the solution gives us  $m = \text{lcm}(60, 2 \times 84) = 840$

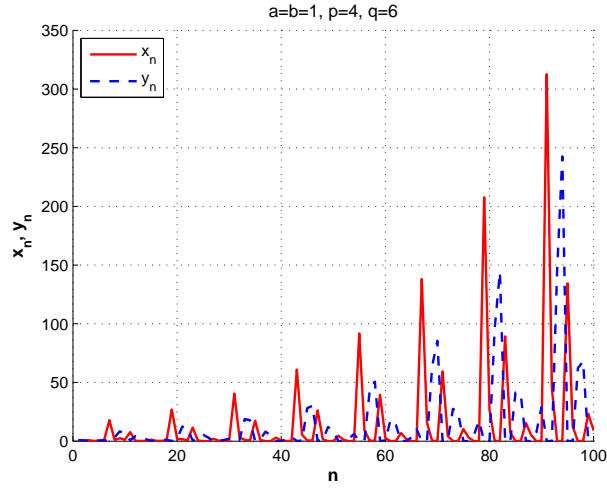


FIGURE 5. The above figure illustrates the behavior of a particular solution of the system

$$x_n = \frac{1}{y_{n-4}}, \quad y_n = \frac{y_{n-4}}{x_{n-6}y_{n-6}}, \quad n = 0, 1, \dots$$

Evidently, that particular solution of the given system behaves accordingly to Theorem 1–(iii)

## REFERENCES

- [1] Cinar, C., On the positive solutions of the difference equation system  $x_{n+1} = \frac{1}{y_n}, y_{n+1} = \frac{y_n}{x_{n-1}y_{n-1}}$ , *Appl. Math. Comp.*, **158** (2004), pp. 303–305.
- [2] Cinar, C., Yalçinkaya, I., On the positive solutions of the difference equation system  $x_{n+1} = \frac{1}{z_n}, y_{n+1} = \frac{1}{x_{n-1}y_{n-1}}, z_{n+1} = \frac{1}{x_{n-1}}$ , *Int. Math. J.*, **5** (2004), pp. 517–519.
- [3] Cinar, C., Yalçinkaya, I., On the positive solutions of the difference equation system  $x_{n+1} = \frac{1}{z_n}, y_{n+1} = \frac{x_n}{x_{n-1}}, z_{n+1} = \frac{1}{x_{n-1}}$ , *Int. Math. J.*, **5** (2004), pp. 525–527.
- [4] Cinar, C., Yalçinkaya, I., On the positive solutions of the difference equation system  $x_{n+1} = \frac{1}{z_n}, y_{n+1} = \frac{y_n}{x_{n-1}y_{n-1}}, z_{n+1} = \frac{1}{x_n}$ , *J. Inst. Math. Comp. Sci.*, **18** (2005), pp. 91–93.
- [5] Cinar, C., Yalçinkaya, I., Karatas, R., On the positive solutions of the difference equation system  $x_{n+1} = \frac{m}{y_n}, y_{n+1} = \frac{py_n}{x_{n-1}y_{n-1}}$ , *J. Inst. Math. Comp. Sci.*, **18** (2005), pp. 135–136.
- [6] Irićanin, B. D., Liu, W., On a higher-order Difference Equation *Disc. Dyn. Nat. Soc.*, **2010** (2010), Article ID 891564, 6 pages.
- [7] Grove, E.A. and Ladas, G., Periodicities in Nonlinear Difference Equations, Chapman and Hall/CRC, 2005.
- [8] Özban, A. Y., On the positive solutions of the system of rational difference equations  $x_{n+1} = \frac{1}{y_{n-k}}, y_{n+1} = \frac{y_n}{x_{n-m}y_{n-m}}$ , submitted for publication.
- [9] Özban, A. Y., On the positive solutions of the system of rational difference equations  $x_{n+1} = \frac{1}{y_{n-k}}, y_{n+1} = \frac{y_n}{x_{n-m}y_{n-m-k}}$ , *J. Math. Anal. Appl.*, **323** (2006), pp. 26–32.
- [10] Özban, A. Y., On the system of rational difference equations  $x_{n+1} = \frac{a}{y_{n-3}}, y_{n+1} = \frac{by_{n-3}}{x_{n-q}y_{n-q}}$ , *Appl. Math. Comp.*, **188**(1) (2007), pp. 833–837.
- [11] Yang, Y., Chen, L., and Shi, Y.-G., On solutions of a system of rational difference equations, *Acta Math. Univ. Comenianae*, Vol. LXXX, 1 (2011), pp. 63–70.
- [12] Yang, X., Liu, Y., Bai, S., On the system of high order rational difference equations  $x_n = \frac{a}{y_{n-p}}, y_n = \frac{by_{n-p}}{x_{n-q}y_{n-q}}$ , *Appl. Math. Comp.*, **171**(2) (2005), pp. 853–856.

JULIUS FERGY TIONGSON RABAGO, DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE, COLLEGE OF SCIENCE, UNIVERSITY OF THE PHILIPPINES BAGUIO, BAGUIO CITY 2600, BENGUET, PHILIPPINES

*E-mail address:* jfrabago@gmail.com